Indian Statistical Institute, Bangalore Centre

B.Math (Hons) II Year, Second Semester

Mid-Sem Examination

Optimization

Time: 3 Hours | March 06, 2013

Maximum marks : 30

Instructor: Pl.Muthuramalingam

1. $A = [\underset{\sim}{a}^1, \underset{\sim}{a}^2, \cdots, \underset{\sim}{a}^n]$ be matrix with column vectors $\underset{\sim}{a}^i$ in $R^{m_0}_{col}$ where $m_0 \leq n$. Let $\underset{\sim}{b} \in R^{m_0}_{col}$. Assume that $P = \{\underset{\sim}{x} \in R^n_{col} : \underset{\sim}{x} \geq \underset{\sim}{0}, A \ \ \underset{\sim}{x} = \underset{\sim}{b}\}$ is nonempty.

   For any $\underset{\sim}{x} \in P$, let supp $\underset{\sim}{x} = \{i : x_i \neq 0\}$, where $\underset{\sim}{x} = (x_1, x_2, \cdots, x_n)^t$. Show that $\underset{\sim}{x}$ is an extreme point of $P \Leftrightarrow \{\underset{\sim}{a}^i : i \in \text{supp } \underset{\sim}{x}\}$ is a linearly independent set. [6]

2. Show that dual(dual)=primal problem, by using the primal dual table. [4]

3. Let$A$ be real symmetric $n \times n$ square matrix, $A = ((a_{ij}))\ i, j = 1, 2, \cdots, n$. Let $A_1, A_2, \cdots, A_k, \cdot\ ; A_n$ be the leading diagonal matrix given by $A_k = ((a_{ij}))\ i, j = 1, 2, \cdots, k$. If det $A_k > 0$ for each k, then show that $\underset{\sim}{u}' A \underset{\sim}{u} \geq 0$ for all $\underset{\sim}{u}$.

   Hint: Consider TAT' or $T'AT$ where $T = \begin{bmatrix} I_{n-1} & 0 \\ \underset{\sim}{y} & 1 \end{bmatrix}$ for a suitable $\underset{\sim}{y} \in R^{n-1}_{row}$ [5]

4. Let $R^n_{++} = \{\underset{\sim}{x} \in R^n : \underset{\sim}{x} = (x_1, x_2, \cdots, x_n), x_i \underset{\neq}{>} 0 \text{ for each } i\}$. Let $f(\underset{\sim}{x}) = -\log(x_1^{d_1}\ x_2^{d_2}\ x_n^{d_n})$ for $\underset{\sim}{x}$ in R_{++} where $d_i > 0$ for each i. Is f a convex function? If so prove it. [3]

   Hint: a) Is the proof obvious for $n = 1$?

   b) Is the sum of convex functions convex?. If so prove it.

5. a) Find the maximum and minimum of $f(x, y) = x^2 - y^2$ on the unit circle $g(x, y) = x^2 + y^2 - 1 = 0$ using Lagrange's multipliers method. [3]

   b) Do the same using the substitution $x = \cos\theta, y = \sin\theta$. [1]

6. State Kuhn-Tucker theorem. [3]

7. Let $P = \{\underset{\sim}{x} \in R^n_{col} : \underset{\sim}{x} \geq \underset{\sim}{0}, A\ \ \underset{\sim}{x} = \underset{\sim}{b}\}$ be nonempty set and $\underset{\sim}{d}$ any non zero extremal direction for P. Let $\underset{\sim}{c} \in R^n_{col}$ be cost vector. Let $f(\underset{\sim}{x}) = \underset{\sim}{c}'\underset{\sim}{x}$ be bounded below on P. One of $\underset{\sim}{c}'\ \underset{\sim}{d}\ \geq 0, \underset{\sim}{c}'\ \underset{\sim}{d} < 0$ is true. Guess the correct answer and prove it. [2]

8. Determine the value of the parameter d such that the feasible set determined by

$$\begin{aligned} x_1 + x_2 + x_3 &\leq d \\ x_1 + x_2 - x_3 &= 1 \\ 2x_3 &\geq d \end{aligned}$$

is empty. [3]

	Primal	Dual Table
	$A, \mathbf{x}, \mathbf{b}, \mathbf{c}$	$A^t, \mathbf{y}^t, \mathbf{c}^t, \mathbf{b}^t$
$i\varepsilon I_1,$	$\sum_j a_{ij}x_j = b_i$	$y_i real, y_i \gtrless 0$
$i\varepsilon I_2,$	$\sum_j a_{ij}x_j \geq b_i$	$y_i \geq 0$
$i\varepsilon I_3,$	$\sum_j a_{ij}x_j \leq b_i$	$y_i \leq 0$
$j\varepsilon J_1,$	$x_j real, x_j \gtrless 0$	$\sum_i y_i a_{ij} = c_j$
$j\varepsilon J_2,$	$x_j \geq 0$	$\sum_i y_i a_{ij} \leq c_j$
$j\varepsilon J_3,$	$x_j \leq 0$	$\sum_i y_i a_{ij} \geq c_j$
	$min \sum_j c_j x_j$	$max \sum_i y_i b_i$